Differential Equations and Civic Engagement

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"What are some problems facing the world today?"

This provocative question has become the standard opening gambit in my math teaching. Students' responses include: climate change, terrorism, HIV/AIDS, Asian flu, energy dependence, over population, animal extinctions and pollution. I go on to explain that a major goal of our math course will be to see how mathematics can be used to address these important societal issues. All to often, mathematics courses focus exclusively on the mathematical content without making linkages to such larger issues.

In his book *What the Best College Teachers Do*, Ken Bain observes that highly successful teachers often start out their courses by painting the broadest possible picture of the importance of what they are going to teach so as to stimulate student interest and motivation. As their courses progress, they continue to show the broader implications of what they are teaching. Many Quantitative Literacy courses, courses that aim to teach basic quantitative skills to the general college student, incorporate this best practices approach by teaching mathematics in the context of meaningful real-world problems. Here I describe ways that I make such real-world linkages in more advanced mathematics courses: calculus and particularly differential equations. In these efforts, I make use of the book *Collapse: How Societies Choose to Fail or Succeed* by Jared Diamond. The book examines human societies throughout history that have died out, the factors that led to their collapses, and the lessons we might learn to prevent a collapse of our present day global society.

The differential equations course is taught to between fifteen and twenty sophomore, junior and senior math and science majors. I use the text *Differential Equations* by Blanchard, Devaney and Hall. Over the past several years I have been focusing the course more on mathematical modeling than on physics and engineering applications. Since Bryn Mawr is a liberal arts college without an engineering program and our physics department teaches its own mathematical methods course, I have the freedom to replace some traditional topics with material on modeling.

The first time I used the Collapse book, students were assigned to read the entire book, a chapter or two per week and write a page long reaction paper for each reading that responded to the prompt: "How does the mathematics we have been learning relate to the topics discussed in the book?" Students were able to find connections for every chapter in the book. At the end of course survey, though, the majority of students reported that they felt reading the entire book was too time consuming when combined with their other assignments in the course. This past year I had the students read a few select chapters that had particularly strong links with important topics in the course, specifically various aspects of population modeling. The student evaluations for this modified approach were nearly uniformly positive.

The first model of population growth that we study involves the exponential function. To help my calculus and differential equations students appreciate that even this simple model can have dramatic implications, I have them read the chapter *Malthus in Africa: Rwanda's Genocide* in which Diamond argues that an important contributing factor to the genocide was over population. I then give them an assignment that was developed with the assistance of Wen Gao, a Bryn Mawr math major, and was inspired by our participation at the 2006 Mathematics of Social Justice conference at Lafayette College. Using data from the chapter and from international population websites, students are asked to calculate for Rwanda the growth rate of population in the decades before the genocide and the population doubling time and then predict what the population will be in later years. For the years after the genocide, they find that their predications significantly overestimate the actual population and are asked to account for the discrepancy. They realize that their overestimates are due to the deaths of hundreds of thousands of people during the genocide period and face the sobering fact that numbers arising from mathematical calculations can have a very human dimension.

A topic that I have made a particular focus of my differential equations course is modeling population growth where the population being studied also undergoes harvesting. As an illustrative example, imagine fishermen in the Grand Banks region near Newfoundland who each year harvest (catch) some amount of the fish population. To start with, there are a certain number of fisherman involved who each year catch roughly a constant amount of fish. Should we allow more fishermen, perhaps equipped with sophisticated fishing technology, to join the hunt? A reasonable response might be that, to avoid the danger of over-fishing, we could allow a small number of additional fisherman to join in. We expect that such a change would increase the catch by a relatively small amount and hence decrease, by a similarly moderate amount, the level of fish remaining in the Grand Banks. However it turns out that such that such a seemingly reasonable strategy can be dangerously misguided.

Mathematically, one can model population growth with harvesting via a differential equation of the form: $dP/dt = kP(1 - P/N) - \lambda$, where $P(t)$ is the population, k is the growth rate, N is the carrying capacity and λ is the harvesting level. A study of the solutions of this equation for various harvesting levels shows the existence of a critical fishing level; technically, it is called the bifurcation value. If the fishing level is increased beyond this critical value, even very slightly, then the model predicts that there will be a drastic crash in the fish population, potentially leading to extinction or near extinction.

The moral of the story is that, if one happens to be unlucky enough to be close to the critical harvesting value, then even a small additional increase in the harvesting level can have cataclysmic implications for the population. Thus great care needs to be taken when increasing harvesting levels even by small amounts, least we inadvertently cause a population crash. Here is an example where mathematics provides us with a key insight that runs counter to our natural intuition.

Sadly, the implications of a critical harvesting level have not been well understood and acted upon: there are numerous examples in the world of fisheries that have been over-fished leading to a precipitous decline in the fish stock. Such situations continue today and are regularly in the news.

The phenomena of over-harvesting is not limited to fishing situations. In its general form, it is often referred to as the "tragedy of the commons". Consider a community whose citizens let their sheep graze on a shared tract of land, the commons. In this situation, no one individual has any incentive to limit the amount of grazing done by his sheep. Over time, the commons will become depleted of grass and cease to be usable for grazing. In the language of our previous example, over-harvesting has caused the population of grass to crash.

To prepare my students to better appreciate the amazing ability of mathematics to explain and predict population crashes, I want them to first experience for themselves how seemingly reasonable human behavior can lead to over-harvesting.

The students read the chapter *Twilight at Easter* that examines the collapse of the society on Easter Island, home to the famous stone statues. They learn that a major factor in the collapse was the complete deforestation of the island and are left to wonder how a society could be so short sighted as to cut down all of its trees. Did no one notice that the tree population was drastically diminishing? Why did no one take steps to address the issue? They feel, a bit smugly, that they would be smarter than the Easter Islanders.

We then have a special three-hour evening meeting of the class in which we play the simulation game Fishing Banks, Ltd created by Dennis Meadows. In this game, teams of students manage their own fishing fleets with the goal of maximizing profit. Over time, what invariably happens is that the teams build up large fishing fleets to maximize their short-term profit, over-harvest the fish population and cause the fish stock to crash to extinction. At this point, with no more fish to catch, the fish companies go bankrupt and hence fail to meet their goal of maximizing profit. The population crash happens even though the teams get feedback after each round on the amount of fish they have caught. By the time they notice that the stocks are decreasing, the corrections they make are too little and too late to stop the extinction. As we debrief this experience, the students realize that they have fallen into the same trap as the Easter Islanders: by over-harvesting a valuable resource, they have driven it to extinction.

Now that the students have a visceral understanding of the over-harvesting phenomena, I introduce the differential equation $dP/dt = kP(1 - P/N) - \lambda$, mentioned earlier, that models the situation, and we undertake its mathematical analysis. Students learn that mathematical modeling can be used to predict and explain the population crash phenomena and can thereby serve as a counter-weight to the many pressures encouraging over-harvesting of resources.

We finish the unit with a discussion of the interplay between mathematical modeling and government and business policy making. Why is it that even though modeling can predict negative consequences, as with over-fishing or climate change, it is so hard to get society to take preventative action? Society might better served by leaders with a firm understanding of mathematics in the context of policy making. By including in our math courses components that link mathematics to issues of social relevance, we can prepare and inspire our students to become these future leaders.

Further information about Donnay's differential equations course can be found at his website: www.brynmawr.edu/math/people/donnay .

Biography:
Victor J. Donnay is Professor of Mathematics at Bryn Mawr College, where he employs a variety of creative teaching strategies to better help his students enjoy and learn mathematics. He is also Co-Principal Investigator of the Math Science Partnership of Greater Philadelphia (MSPGP), a collaboration of 13 colleges and universities and 46 school districts that are working together to improve math and science education. Professor Donnay's education course: Changing Pedagogies in Math and Science Education, gives Bryn Mawr students the opportunity to participate in the educational reform activities of the MSPGP.