Bias in prime numbers for elliptic curves over function fields

Abstract: In 1853, Chebyshev noted that primes congruent to 3 modulo 4 tend to predominate over those congruent to 1 modulo 4. This phenomenon, later known as “Chebyshev’s bias”, has been extensively studied and generalized by many number theorists. In particular, Rubinstein and Sarnak’s seminal work in 1994 provided a general framework to quantify the size of this bias. Motivated by this, Mazur proposed to study a similar bias for elliptic curves. If we fix an elliptic curve $E$ over rationals, he compares the number of primes $p$ with $a_p(E) > 0$ against those with $a_p(E) < 0$, where $a_p(E)$ is the usual trace of Frobenius at $p$. In this talk, which is joint work with Daniel Fiorilli and Florent Jouve, we study a function field analogue of the bias for elliptic curves and prove many results which are consistent with the number field case. Some of our results are much more unconditional than their counterparts in the number field setting. We specialize in Ulmer’s family of elliptic curves and show that the biases in this family behave in a variety of ways.