Kloosterman sums in residue classes

Abstract: Distribution of values of complete exponential sums, and prominently of Kloosterman sums, is of central importance in number theory and arithmetic geometry. While Weil’s bound gives the best possible estimate for the size of an individual Kloosterman sum, the distribution of values of Kloosterman sums to varying moduli is intimately related to the spectral theory of automorphic forms.

In this pair of talks, we will present upper bounds for sums of Kloosterman sums against general “arithmetic” weight functions which, in particular, prove power cancellation in sums of Kloosterman sums over arithmetic progressions. This is the first such equidistribution result over non-zero congruence classes, with cancellation of square-root strength in any fixed primitive class up to bounds towards the Ramanujan conjecture. Along the way, we develop an elementary but unexpected device for encoding arithmetic weights using twisted Kloosterman sums as well as a Kuznetsov trace formula with quite general adelic weights.

This is joint work with Valentin Blomer.