Abstract: An important way of analyzing a field $K$ is to understand the polynomials with coefficients in $K$ that do or do not have roots in $K$. This leads naturally to Galois theory. In many circumstances, it is useful and interesting to categorize all Galois extensions of a given field in an explicit way. After reviewing some fields where this problem is easily solved, we will turn our attention to the Laurent series field $K = k((t))$, where $k$ is an algebraically closed field of characteristic $p > 0$. Recent work of the speaker and Rachel Pries has explicitly classified all extensions $L/K$ where $\text{Gal}(L/K)$ is an extension of a prime-to-$p$ group by a cyclic group. We will discuss these results and their relation to algebraic geometry, in particular, the problem of lifting varieties and automorphisms from characteristic $p$ to characteristic 0.