Abstract:

For which positive integers a, b, c (with no common factors) does there exist a solution to the equation \( ax + by + cz = 0 \), where \( x, y, \) and \( z \) are conjugate algebraic numbers (i.e. roots of the same irreducible polynomial)? In the 80's Smyth showed that this problem is equivalent to a very simple question about integer solutions to the equation, and conjectured that there is always a solution as long as \( a, b, \) and \( c \) are pairwise relatively prime, and could be the side lengths of a triangle -- that is, \( a \) is at most \( b+c \), \( b \) is at most \( a+c \), etc. This conjecture remains open. We'll talk about why the conjecture seems difficult, both theoretically and computationally, and show that a stronger version can't be true. This contains joint work in progress with Jennifer Berg (Rice).